ÉRETTSÉGI VIZSGA • 2011. május 3.

MATEMATIKA
ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

NEMZETI ERŐFORRÁS MINISZTÉRIUM

Instructions to examiners

Formal requirements:

1. Mark the paper in **ink, different in colour** from the one used by the candidate. Indicate the errors, incomplete solutions, etc in the conventional way.
2. The first one of the rectangles under each problem shows the maximum attainable score on that problem. The **points** given by the examiner are to be entered **in the rectangle** next to that.
3. **If the solution is perfect**, it is enough to enter the maximum scores in the appropriate rectangles.
4. If the solution is incomplete or incorrect, please indicate the individual **partial scores** in the body of the paper, too.

Assessment of content:

1. The markscheme may contain more than one solution for some of the problems. If the **solution by the candidate is different**, allocate the points by identifying the parts of the solution equivalent to those of the one given in the markscheme.
2. The subtotals in the markscheme can be **further divided**, but the scores awarded should always be whole numbers.
3. If it is clear that the reasoning and the final answer are both correct, you may award the maximum score even if the solution is **less detailed** than the one in the markscheme.
4. If there is a **calculation error** or inaccuracy in the solution, only take off the points for that part where the error is made. If the reasoning remains correct and the error is carried forward, the points for the rest of the solution should be awarded.
5. **In the case of a principal error**, award no points at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the markscheme.) However, if the wrong information obtained owing to the principal error is carried forward to the next section or in the next part of the problem and is used correctly, the maximum score is due for the next part, provided that the error has not changed the nature of the task to be completed.
6. Where the markscheme shows a **remark** or **unit** in brackets, the solution should be considered complete without that remark or unit as well.
7. If there are more than one different approaches to a problem, **assess only the one indicated by the candidate**.
8. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
9. **Do not take off points** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **Assess only four out of the five problems in Section II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted in their total score. Should there be a solution to that problem, it does not need to be marked. However, if it is still not clear which problem the candidate does not want to be assessed, assume automatically that it is the last one in the question paper, and do not assess that problem.

I.

1. a) Solution 1.		
The function *f* is a quadratic function of negative leading coefficient, restricted to a closed interval. Its graph is a parabolic arc that is concave down.	1 point	
Completing the square: $-x^2-2x+3\equiv-(x+1)^2+4$.	1 point	
The point (–1;4) is the vertex of the parabola, and it belongs to the graph of function *f*, too.	1 point	
Thus *f* is strictly increasing on the interval $[-2;-1]$, and strictly decreasing on the interval $[-1;5]$.	1 point	
It follows from the above that *f* has a maximum at –1, and the maximum value is 4.	1 point	
The minimum may lie at either endpoint of the closed interval: $f(-2)=3$, $f(5)=-32$.	1 point	
The function *f* has its minimum at 5, the minimum value is $f(5)=-32$.	1 point	
Total:	**7 points**	

Remarks.

1. If the candidate starts the solution by drawing a graph and uses the graph correctly to determine the intervals of monotonicity and the maximum and minimum, award the 7 points, provided that there is an explanation of the graph being correct.

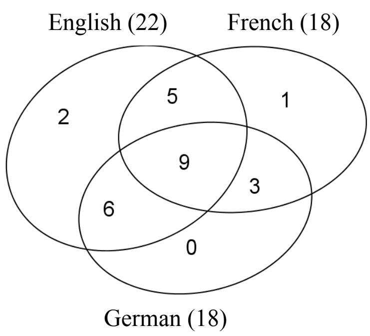
2. If the graph is wrong but the values are read correctly from the graph, award 3 points (1 for monotonicity, 2 for maximum and minimum).

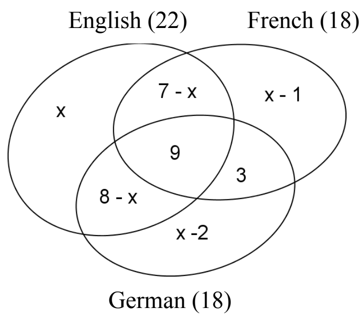
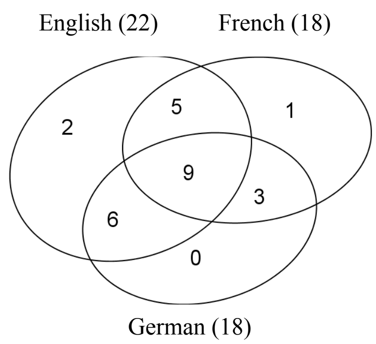
3. $x=-1$ may or may not be included in the intervals of monotonicity: accept either way.

1. a) **Solution 2.**		
The derivative function of the function $x \mapsto -x^2 - 2x + 3$ defined on the set of real numbers is $x \mapsto -2x - 2$.	1 point	
The original function is strictly increasing where the derivative function is positive, and strictly decreasing where it is negative.	1 point	
$-2x - 2 > 0 \Leftrightarrow x < -1$ and $-2x - 2 < 0 \Leftrightarrow x > -1$.	1 point	
The function *f* is a restriction of the function $x \mapsto -x^2 - 2x + 3$ defined on the set of real numbers, therefore it is strictly increasing on the interval $[-2; -1[$, and strictly decreasing on $]-1; 5]$.	1 point	
It follows from the above that *f* has a maximum at –1, and the maximum value is 4.	1 point	
The minimum may lie at either endpoint of the closed interval: $f(-2) = 3$, $f(5) = -32$.	1 point	
The function *f* has its minimum at 5, the minimum value is $f(5) = -32$.	1 point	
Total:	**7 points**	
<u>*Remark.*</u> $x = -1$ *may or may not be included in the intervals of monotonicity: accept either way.*		

1. b)		
The expression $\dfrac{1}{\lg(x^2 + 2x - 3) - \lg 5}$ is meaningful if $x^2 + 2x - 3 > 0$, and	1 point	
$\lg(x^2 + 2x - 3) \neq \lg 5$.	1 point	
The solution of the inequality on the set of real numbers: $x < -3$ or $x > 1$,	1 point	
but since $-2 \le x \le 5$, the condition $1 < x \le 5$ must hold.	1 point	
$\lg(x^2 + 2x - 3) = \lg 5$ is true exactly if $x^2 + 2x - 3 = 5$.	1 point	
The solutions of this equation are – 4 and 2.	1 point	
Thus the expression is meaningful for those real numbers *x* for which $1 < x \le 5$ and $x \neq 2$.	1 point	*The 1 point is due for any correct form of the answer. For example,* $]1; 2[\cup]2; 5]$ *or* $\{x \in \mathbf{R} \mid 1 < x \le 5, x \neq 2\}$.
Total:	**7 points**	

2. a)		
Out of the 12 students with certificates in both German and French, 12 – 3 = 9 also have one in English.	1 point	
9 students answered “yes” to all three questions.	2 points	
Total:	**3 points**	

2. b) Solution 1.		
Out of the 22 students with English certificates, 22 – 9 = 13 have either one or two certificates.	1 point	
Therefore, 13 students belong to the union of those with English certificates who have no German or no French certificate.	1 point	
The numbers of elements in these two sets individually are 7 and 8, and their union has 13 elements. Thus their intersection contains 15 – 13 = 2 elements: these are the students who only have a certificate in English.	2 points	
Using this information, the set diagram below can be filled in with the numbers of elements: 	3 points	
The total number of those having at least one of the three language certificates is 22 + 3 + 1 = 26.	1 point	
(29 – 26 =) 3 students answered “no” to all three questions.	1 point	
Total:	**9 points**	

2. b) **Solution 2.**		
Let x denote the number of those who only have a certificate in English.	1 point	*This point is also due if the idea is only reflected by the solution.*
With this notation, the following Venn diagram can be constructed: 	3 points	*The 3 points are due for filling out the "English" set correctly.*
The number of those with English certificates is 22, so $24 - x = 22$.	1 point	
There are $(x =) 2$ students who only have English certificates.	1 point	
With this value of x, the numbers of elements are as follows: 	1 point	
The total number of those having at least one of the three language certificates is 22 + 3 + 1 = 26.	1 point	
29 – 26 = 3 students have no language certificates at all, that is, 3 students answered "no" to all three questions.	1 point	
Total:	**9 points**	

3. Solution 1.		
If there were *x* kg of apricots in a crate on Monday, and the merchant bought *y* crates,	1 point	*This point is also due if the idea is only reflected by the solution.*
then there were ($x-2$) kg of peaches in a crate on Tuesday, and he bought $(y+8)$ crates altogether.	1 point	
Thus the equations $xy=165$ and $(x-2)(y+8)=165$ must hold.	1 point	
The task is to solve the simultaneous equations $\left.\begin{array}{l} xy=165 \\ (x-2)(y+8)=165 \end{array}\right\}$, where *x* and *y* are positive numbers. By eliminating the brackets in the second equation, the equation $xy-2y+8x-16=165$ is obtained.	1 point	
Since $xy=165$, it follows that $165-2y+8x-16=165$,	1 point	
that is, $4x-y=8$.	1 point	
Hence $y=4x-8$. By substituting $4x-8$ for *y* in the equation $xy=165$,	1 point	
the quadratic equation $4x^2-8x-165=0$ is obtained.	1 point	
The positive root of the equation is $x=7.5$. (The negative solution is –5.5)	1 point	
Hence $y=22$.	1 point	
Calculating with these values, there were 5.5 kg of peaches in a crate on Tuesday and the merchant bought 30 crates of them. (These results agree with the conditions of the problem.)	1 point	*This point is for checking with the wording of the problem.*
Thus on Monday, there were 7.5 kg of apricots in a crate and he bought 22 crates.	1 point	
Total:	**12 points**	

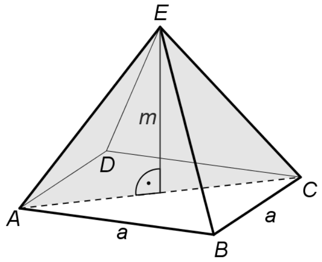
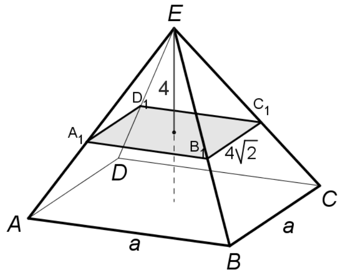
3. Solution 2.		
If the retailer bought n crates of apricots on Monday, then each crate contained $\frac{165}{n}$ kg of apricots.	2 points	
Thus on Tuesday, he bought $(n+8)$ crates of peaches, and each crate contained $\left(\frac{165}{n}-2\right)$ kg of peaches.	2 points	
$(n+8)\cdot\left(\frac{165}{n}-2\right)=165$.	2 points	
Rearranged: $n^2+8n-660=0$.	2 points	
The only positive root is $n=22$ (the other root is $n=-30$).	2 points	
On Monday, there were 7.5 kg of apricots in a crate and the retailer bought 22 crates.	1 point	
These results agree with the conditions of the problem (he bought 30 crates on Tuesday, and there were 5.5 kg of peaches in a crate).	1 point	*This point is for checking with the wording of the problem.*
Total:	**12 points**	

Remark.

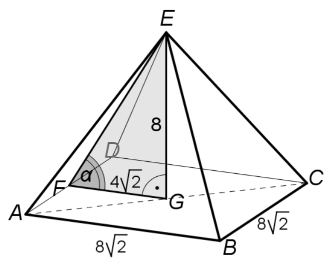
If the amount of apricots in a crate is s kg on Monday, then he bought $\frac{165}{s}$ *crates.*

The equation obtained is then: $(s-2)\cdot\left(\frac{165}{s}+8\right)=165$. *Rearranged:* $4s^2-8s-165=0$.

Roots: $s_1=7.5$ *and* $s_2=-5.5$.

4. a) **Solution 1.**		
The solution uses the notation of the diagrams below.  		
Let *a* denote the base edge and let *m* be the height of the pyramid.	1 point	*This point is due if notations are clearly shown in a diagram.*
$AC = a\sqrt{2}$,	1 point	
and the area of triangle *AEC* is (1) $64 = \frac{a\sqrt{2} \cdot m}{2}$.	1 point	
The plane parallel to the base intersects the pyramid in a square with a side of $\sqrt{32}\ \left(= 4\sqrt{2}\right)$ cm.	1 point	
Because of the (central) similitude of the squares *ABCD* és $A_1B_1C_1D_1$ (or of the two pyramids of apex *E*), the lengths of the corresponding line segments are proportional:	1 point	
$\frac{a}{4\sqrt{2}} = \frac{m}{4}$,	1 point	
$m = \frac{a}{\sqrt{2}}$.	1 point	
By substituting in equation (1) (expressing the area of triangle *AEC*): $64 = \frac{a^2}{2}$, $a^2 = 128$.	1 point	
The area of the base of the pyramid is 128 cm^2.	1 point	
$a^2 = 128 \Rightarrow a = 8\sqrt{2}$ (since $a > 0$), the height of the pyramid is $m = \frac{a}{\sqrt{2}} = 8$ (cm).	1 point	
Total:	**10 points**	

4. a) **Solution 2.**		
Let a denote the base edge and let m be the height of the pyramid.	1 point	*This point is due if notations are clearly shown in a diagram.*
$AC = a\sqrt{2}$,	1 point	
the area of triangle AEC is (1) $64 = \frac{a\sqrt{2} \cdot m}{2}$.	1 point	
The intersection with a plane parallel to the base (a square) is (centrally) similar to the base.	1 point	
Using the known relationship between areas of similar plane figures: $\frac{a^2}{32} = \frac{m^2}{16}$,	1 point	
that is, $\frac{a^2}{2} = m^2$.	1 point	
(Since $a > 0$ and $m > 0$,) it follows that $m = \frac{a}{\sqrt{2}}$.	1 point	
By substituting in equation (1) (expressing the area of triangle AEC): $64 = \frac{a^2}{2}$, $a^2 = 128$.	1 point	
The area of the base of the pyramid is 128 cm^2.	1 point	
$a^2 = 128 \Rightarrow a = 8\sqrt{2}$ (since $a > 0$), the height of the pyramid is $m = \frac{a}{\sqrt{2}} = 8$ (cm).	1 point	
Total:	**10 points**	

4. b)		
		
If the midpoint of edge AD is F and the centre of the square $ABCD$ is G, the angle in question is $\alpha = \sphericalangle EFG$.	1 point	
In the right-angled triangle EGF, $\tan\alpha = \frac{8}{4\sqrt{2}} = \sqrt{2}$,	1 point	
$\alpha \approx 54.7^\circ$.	1 point	
Total:	**3 points**	

II.

5. a)		
$a_1 = 1 + \frac{\sqrt{3}}{2}$	1 point	
$a_2 = 2 + \frac{\sqrt{3}}{2}$	1 point	
$a_3 = 3$	1 point	
Total:	**3 points**	

5. b) Solution 1.		
For any $\alpha \in [0; 2\pi]$, $a_1 = 1 + \sin\alpha$, $a_2 = 2 + \sin 2\alpha$, $a_3 = 3 + \sin 3\alpha$.	1 point	
These are consecutive terms of an arithmetic progression if $a_1 + a_3 = 2a_2$,	1 point	*The 2 points are due for correctly applying any definition of the arithmetic progression.*
that is, $4 + \sin\alpha + \sin 3\alpha = 2 \cdot (2 + \sin 2\alpha)$.	1 point	
Rearranged: $\sin 3\alpha + \sin\alpha = 2\sin 2\alpha$.	1 point	
With the identity $\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2} \cdot \cos\frac{\alpha-\beta}{2}$ applied to the left-hand side,	1 point	
$2\sin 2\alpha \cdot \cos\alpha = 2\sin 2\alpha$.	1 point	
All terms arranged on the same side and factorised: $\sin 2\alpha \cdot (\cos\alpha - 1) = 0$.	1 point	
(The product on the left-hand side is 0 exactly if one of the factors is 0.) On the set of real numbers, $\sin 2\alpha = 0$ exactly if $2\alpha = k\pi$, that is, if $\alpha = k \cdot \frac{\pi}{2}$ ($k \in \mathbf{Z}$).	1 point	
Since $\alpha \in [0; 2\pi]$, the possible values of α are 0; $\frac{\pi}{2}$; π; $\frac{3\pi}{2}$; 2π.	1 point	*This point is only due if all five correct values are listed.*
On the interval in question, $\cos\alpha = 1$ is only true for $\alpha = 0$ and $\alpha = 2\pi$, These values were both obtained above, in the investigation of the other factor.	1 point	
If $\alpha = 0$, $\alpha = \pi$ or $\alpha = 2\pi$, then $a_1 = 1$, $a_2 = 2$, $a_3 = 3$;	1 point	
if $\alpha = \frac{3\pi}{2}$, then $a_1 = 0$, $a_2 = 2$, $a_3 = 4$, Thus these four values of α provide solutions.	1 point	
$\alpha = \frac{\pi}{2}$ does not give a solution for the problem since in that case $a_1 = a_2 = a_3 = 2$.	1 point	
Total:	**13 points**	

5. b) **Solution 2.**		
For any $\alpha \in [0; 2\pi]$, $a_1 = 1 + \sin\alpha$, $a_2 = 2 + \sin 2\alpha$, $a_3 = 3 + \sin 3\alpha$.	1 point	
These are consecutive terms of an arithmetic progression if $a_1 + a_3 = 2a_2$,	1 point	*The 2 points are due for correctly applying any definition of the arithmetic progression.*
that is, $4 + \sin\alpha + \sin 3\alpha = 2 \cdot (2 + \sin 2\alpha)$.	1 point	
Rearranged: $\sin 3\alpha + \sin\alpha = 2\sin 2\alpha$.	1 point	
Applying the identities $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$ and $\sin 2\alpha = 2\sin\alpha\cos\alpha$:	1 point	
$4\sin\alpha - 4\sin^3\alpha = 4\sin\alpha\cos\alpha$.	1 point	
Hence $\sin\alpha \cdot (1 - \sin^2\alpha - \cos\alpha) = 0$.	1 point	
(The product on the left-hand side is 0 exactly if one of the factors is 0.) On the interval in question, $\sin\alpha = 0$ is true exactly if $\alpha = 0$ or $\alpha = \pi$ or $\alpha = 2\pi$.	1 point	*This point is only due if all three correct values are listed.*
Since $1 - \sin^2\alpha = \cos^2\alpha$, the other factor on the left-hand side can be written in the form $\cos^2\alpha - \cos\alpha = \cos\alpha \cdot (\cos\alpha - 1)$, and it is 0 exactly if $\cos\alpha = 0$ or $\cos\alpha = 1$.	1 point	
Only the equation $\cos\alpha = 0$ adds further values to the values of α found above: $\alpha = \frac{\pi}{2}$ and $\alpha = \frac{3\pi}{2}$.	1 point	
If $\alpha = 0$, $\alpha = \pi$ or $\alpha = 2\pi$, then $a_1 = 1$, $a_2 = 2$, $a_3 = 3$;	1 point	
if $\alpha = \frac{3\pi}{2}$, then $a_1 = 0$, $a_2 = 2$, $a_3 = 4$, Thus these four values of α provide solutions.	1 point	
$\alpha = \frac{\pi}{2}$ does not give a solution for the problem since in that case $a_1 = a_2 = a_3 = 2$.	1 point	
Total:	**13 points**	

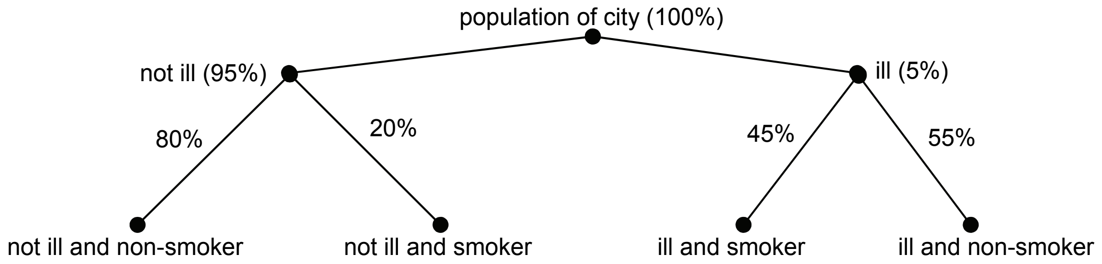
6. a)		
$3^5 = 243$ different draws are possible (each with the same probability).	1 point	
The numbers of blue and red balls drawn are equal if they are both 0, both 1 or both 2.	1 point	*This point is also due if the idea is only reflected by the solution.*
In the first case (0 blue, 0 red), all five balls are white: that is possible in 1 single way.	1 point	
Second case: there are 1 red, 1 blue and 3 white balls. The number of possibilities is $\frac{5!}{3!}\left(=2\cdot\binom{5}{3}\right)=20$. Third case: 2 red, 2 blue and 1 white. The number of possibilities is $\frac{5!}{2!\,2!}\left(=\binom{5}{1}\binom{4}{2}\right)=30$.	3 points	
Thus the number of favourable cases is $1+20+30=51$.	1 point	
The probability of the same number of blue and red balls drawn is $\frac{51}{243}\left(\approx 0.2098\approx 0.21\right)$.	1 point	
Total:	**8 points**	

6. b) Solution 1.		
There are three cases: the number of red and blue balls is equal, there are more red than blue, or more blue than red.	2 points	*These points are also due for a correct but less detailed explanation.*
Since the number of balls of each colour in the urn is the same,	1 point	
the probability of drawing more red than blue equals the probability of drawing more blue than red.	2 points	
Therefore the probability of drawing more blue than red is $\frac{1}{2}\cdot\left(1-\frac{51}{243}\right)=$	2 points	
$\frac{96}{243}\left(\approx 0.95\right)$.	1 point	
Total:	**8 points**	

6. b) **Solution 2.**		
This solution counts directly the number of draws out of the 243 equally probable cases that result in more blue than red balls. The possible numbers of balls are listed below. The first number is the number of blue balls, followed by the number of red balls and then the number of white balls: (1,0,4), (2,0,3), (3,0,2), (4,0,1), (5,0,0), (2,1,2), (3,1,1), (4,1,0), (3,2,0).	1 point	
Since the number of balls of each colour in the urn is the same, the following possibilities out of those listed above are equally probable: (1,0,4), (4,0,1), (4,1,0) (2,0,3), (3,0,2), (3,2,0) (5,0,0) (3,1,1) (2,1,2)	1 point	
The first three may occur in a total of $3\cdot\frac{5!}{4!}\left(=3\cdot\binom{5}{1}\right)=15$ ways,	1 point	
the second three in $3\cdot\frac{5!}{3!2!}\left(=3\cdot\binom{5}{2}\right)$=30 ways,	1 point	
(5,0,0) in 1 way, and (3,1,1) may occur in $\frac{5!}{3!}\left(=2\cdot\binom{5}{3}\right)$ =20 ways.	1 point	
Finally, (2,1,2) may occur in $\frac{5!}{2!\cdot 2!}=\frac{120}{4}=30$ ways.	1 point	
So the number of favourable cases is $15+30+1+20+30=96$.	1 point	
Hence the probability in question is $\frac{96}{243}\ (\approx 0.395)$.	1 point	
Total:	**8 points**	*Award at most 5 points if the numbers of all cases and favourable cases are calculated inconsistently.*

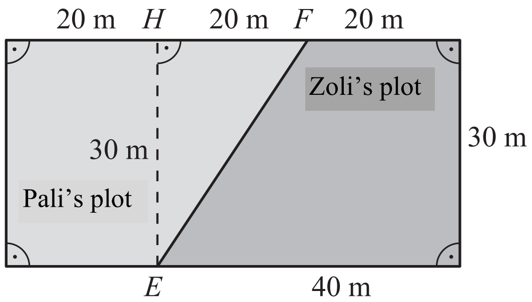
<u>Remark.</u>
If the candidate starts by calculating the probability of more blue than red balls drawn, and then uses the reasoning of Solution 1 of part b) to calculate the probability of an equal number of blue and red, allocate the points according to Solution 1 of part b).

7. a)		
We can use the binomial distribution of parameters $n = 100$ and $p = 0.05$.	1 point	*The point is also due if the solution is based on this idea.*
The probability of no one being ill out of the 100 persons selected is 0.95^{100},	1 point	
The probability of 1 of them being ill is $\binom{100}{1} \cdot 0.05 \cdot 0.95^{99}$.	1 point	
The probability that at most one of the 100 persons is ill with the new disease is $0.95^{100} + \binom{100}{1} \cdot 0.05 \cdot 0.95^{99} \approx$	1 point	
$\approx (0.0059 + 0.0312) \approx 0.0371$.	1 point	
We need (the probability of the complementary event, that is) the probability of at least two of the 100 persons being ill, which is $\approx 1 - 0.0371 \approx 0.9629$.	1 point	
The probability in question, rounded to two decimal places, is 0.96.	1 point	
Total:	**7 points**	

7. b) **Solution 1.**		
It is useful to represent the data in a diagram. 		
(Setting up a model: the population of the city is divided into 4 disjoint groups based on the given information.)	1 point	*The point is also due if the solution is based on this idea.*
The percentage of the population of the city that belongs to each of the four groups is calculated. Not ill and non-smoker: $0.95 \cdot 0.8 = 0.76$, that is, 76%; not ill and smoker: $0.95 \cdot 0.2 = 0.19$, that is, 19%;	1 point	
ill and smoker: $0.05 \cdot 0.45 = 0.0225$, that is, 2.25%; ill and non-smoker: $0.05 \cdot 0.55 = 0.0275$, that is, 2.75%.	1 point	
The percentage of all smokers in the city is (19 + 2.25 =) 21.25%, and among them, 2.25% of the whole population are ill.	1 point	
Hence the percentage of smokers having the disease is $\frac{2.25}{21.25} \cdot 100\%$.	1 point	
Rounded to one decimal place, this is 10.6%.	1 point	
The percentage of all non-smokers in the city is (76 + 2.75=) 78.75%, and among them, 2.75% of the whole population are ill.	1 point	
Hence the percentage of non-smokers having the disease is $\frac{2.75}{78.75} \cdot 100\%$.	1 point	
Rounded to one decimal place, this is 3.5%.	1 point	
Total:	**9 points**	

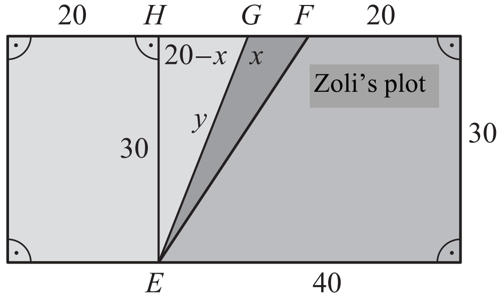
7. b) Solution 2.		
We will calculate with 100 000 inhabitants, since the proportions are independent of the size of the sample.	2 points	
Out of 100 000 inhabitants, 5 000 become ill, 95 000 stay healthy.	1 point	
Out of the 5000 becoming ill, 2250 are smokers, 2750 are non-smokers.	1 point	
Out of the 95 000 staying healthy, 19 000 are smokers, 76 000 are non-smokers.	1 point	
There are 2250 persons having the disease out of 21 250 smokers,	1 point	
this is 10.6% of the smokers.	1 point	
There are 2750 persons having the disease out of 21 250 non-smokers,	1 point	
this is 3.5% of the non-smokers.	1 point	
Total:	**9 points**	

8. a)



Understanding how the plot is divided (e.g. correct diagram).	1 point	
Specifying the right-angled triangle (*EFH*) of hypotenuse *EF* and legs 20 and 30 .	1 point	
Applying the Pythagorean theorem: $EF^2 = 20^2 + 30^2$	1 point	
$EF \approx 36.1$ metres.	1 point	
Total:	**4 points**	

8. b) Solution 1.



Understanding the problem (e.g. correct diagram). (With notations $FG = x$ and $EG = y$:)	1 point	
The area T of triangle EFG is $T = 15x$ (m^2);	1 point	
the value of this area added to Zoli's plot is $30\,000 \cdot 15x$ (forints).	1 point	
The new fence length can be calculated from the right-angled triangle EHG. $FG = 20 - x$.	1 point*	
Applying the Pythagorean theorem: $y^2 = (20-x)^2 + 30^2$	1 point*	
$0 < y = \sqrt{1300 - 40x + x^2}$.	1 pont*	
The cost of building a fence of length EG is $15\,000 \cdot \sqrt{1300 - 40x + x^2}$.	1 point	
The deal is to Zoli's advantage, therefore $15\,000 \cdot \sqrt{x^2 - 40x + 1300} < 30\,000 \cdot 15x$, that is	1 point	
$\sqrt{x^2 - 40x + 1300} < 30x$ (where x is positive, and it expresses in metres the length in question.) (Since each side is non-negative, their squares are in the same relation.) $x^2 - 40x + 1300 < 900x^2$. Hence $0 < 899x^2 + 40x - 1300$ (where x is positive).	1 point	
The only positive zero of the quadratic function $x \mapsto 899x^2 + 40x - 1300$ ($x \in \mathbf{R}$) is ≈ 1.18. (The other zero is ≈ -1.22)	1 point	
The quadratic function above is strictly increasing on the set of positive numbers.	1 point	
Since $1.18 \text{ m} \approx 1.2 \text{ m}$, the length of line segment FG is at least 1.2 m (and at most 8 m).	1 point	
Total:	**12 points**	

Remark.

*Side FG can also be calculated with the cosine rule (for the points marked with *):*

With the notation $\sphericalangle GFE = \alpha$, $\tan\alpha = 1.5$, *that is,* $\alpha \approx 56.31^\circ$ *1 point*

Applying the cosine rule to side EG of triangle EFG: *1 point*

$y = \sqrt{x^2 + 36.06^2 - 72.2x\cos 56.3^\circ}$ *1 point*

8. b) Solution 2.		
It is enough to seek the favourable points *G* on the line segment *FH*.	1 point	
With the notation $x = FG$, as x increases, the length of line segment *EG* strictly decreases (since one leg of the right-angled triangle *EHG* decreases while the other leg stays constant 30 m).	1 point	
The area of triangle *EFG* strictly decreases (since side *FG* increases and the corresponding height does not change).	1 point	
It follows that it is enough to investigate when the cost of fence building will equal the value of the piece of plot received in return.	1 point	
The area received (with x measured in metres) is $\frac{30x}{2} = 15x\ (\mathrm{m}^2)$,	1 point	
and its value is $30\,000 \cdot 15x$ (forints).	1 point	
The length of the fence (from the Pythagorean theorem): $\sqrt{1300 - 40x + x^2}$,	1 point	
the cost of building the fence: $15000 \cdot \sqrt{1300 - 40x + x^2}$.	1 point	
$15000 \cdot \sqrt{x^2 - 40x + 1300} = 30000 \cdot 15x$, that is	1 point	
$\sqrt{x^2 - 40x + 1300} = 30x$ (where x is positive), $x^2 - 40x + 1300 = 900x^2$. Hence $899x^2 + 40x - 1300 = 0$.	1 point	
The positive root is $x \approx 1.18$.	1 point	
Thus the line segment *FG* is at least 1.2 m (and at most 8 m) long.	1 point	
Total:	**12 points**	

8. b) Solution 3.		
This solution uses the requirement of giving the answer to the nearest tenth of a metre (i.e. to the nearest whole decimetre).		
The length of the fence to be built is at least $\sqrt{30^2+12^2}\approx 32.3$ m and at most $EF\approx 36.1$ m.	2 points	
Zoli spends at least $32.3\cdot 15\,000 = 484\,500$ forints and at most $36.1\cdot 15\,000 = 541\,500$ forints on the fence.	1 point	
The cost of the fence thus equals the value of at least 16.15 m^2 and at most 18.05 m^2 of land.	1 point	
With the notation $FG = x$, if $\frac{x\cdot 30}{2} > 18.05$, then the deal will certainly favour Zoli.	1 point	
Hence (because of the rounding), $x \geq 1.3$ (m) is obtained.	1 point	
The distance FG corresponding to 16.15 m^2: $\frac{2\cdot 16.15}{30}\approx 1.08$ (m). (It follows from monotonicity that Zoli will be disadvantaged by any smaller x than that.)	1 point	
All there remains to investigate is whether 1.1 m or 1.2 m is also favourable for Zoli.	1 pont	
(table below)	2 points	
The table shows that 1.2 is also favourable for Zoli.	1 point	
Summarized: If FG is at least 1.2 m (and at most 8 m), the deal will be to Zoli's advantage.	1 point	
Total:	**12 points**	

FG (m)	1.1	1.2
cost of fence (forints)	531 857	531 059
value of land received (forints)	495 000	540 000
money gained by Zoli (forints)	–36 857	+8941

9. a)		
The daily profit of the factory in the case of n sets manufactured is $p(n) = 18n - 0.2 \cdot n^{1.5} - 12n - 300 =$	1 point	
$= -0.2 \cdot n^{1.5} + 6n - 300.$	1 point	
The function $f: \mathbf{R}^+ \to \mathbf{R};\ f(x) = -0.2 \cdot x^{1.5} + 6x - 300$ is differentiable and $f'(x) = -0.3 \cdot x^{0.5} + 6$.	1 point	*Do not award his point if the function p is differentiated.*
$f'(x) = 0$ is necessary for a maximum or minimum of f to exist.	1 point	
$-0.3 \cdot x^{0.5} + 6 = 0 \Leftrightarrow x = 400$.	1 point	
Since $f''(x) = -0.15 \cdot x^{-0.5} = -\frac{0.15}{\sqrt{x}} < 0$,	1 point	*These 2 points are also due if the maximum is justified by reference to a sign change of the first derivative (positive to negative) at x = 400.*
the daily profit is maximum for $x = 400$ since the maximum of p occurs at the same point (because the maximum of f belongs to the domain of p, too).	1 point	
The profit will be maximum if 400 sets are manufactured a day.	1 point	
The maximum profit is $p(400) = -0.2 \cdot 400^{1.5} + 6 \cdot 400 - 300 = 500$ (euros).	1 point	
Total:	**9 points**	

9. b)		
The volume of the original cube is $V_c = 27\ \text{cm}^3$.	1 point	
A right-angled tetrahedron is cut off at each vertex. Three faces of the tetrahedron are congruent isosceles right-angled triangles of leg 1 cm, pairwise perpendicular to each other.	2 points	*These 2 points may be awarded for a neat diagram if the data are shown.*
With any of these faces considered base, the height of the tetrahedron is 1 cm.	1 point	
The total volume of the 8 tetrahedra cut off at the 8 vertices is $V = 8 \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{4}{3}$ (cm^3).	1 point	
The volume of the remaining solid is $V_c - V = 25\frac{2}{3}\left(=\frac{77}{3}\text{cm}^3\right)$.	1 point	
Thus $\frac{V_c - V}{V_c} = \frac{77}{81} \approx 95\%$.	1 point	
Total:	**7 points**	